RISK, AMBIGUITY, AND THE SAVAGE AXIOMS: COMMENT

By HOWARD RAIFFA

Ellsberg writes:

> I propose to indicate a class of choice-situations in which many otherwise reasonable people neither wish nor tend to conform to the Savage postulates, nor to the other axiom sets that have been devised. But the implications of such a finding, if true, are not wholly destructive. First, both the predictive and normative use of the Savage or equivalent postulates might be improved by avoiding attempts to apply them in certain, specifiable circumstances where they do not seem acceptable. Second, we might hope that it is precisely in such circumstances that certain proposals for alternative decision rules and nonprobabilistic descriptions of uncertainty (e.g., by Knight, Shackle, Hurwicz, and Hodges and Lehmann) might prove fruitful. I believe, in fact, that this is the case.[1]

Let me introduce myself as one who accepts Savage's postulates or equivalent rules for a much wider domain of problems than does either Ellsberg or Fellner. In particular, I find that I would want to behave in a manner consistent with Savage's normative prescriptions of behavior in the examples cited by the above authors. At the outset I will agree with them that it is not hard to elicit from most people (and I include myself in this category) a set of mutually inconsistent responses to questions or to observe in their actions inconsistent behavior. But I wish to reaffirm, what these authors also emphasize, that Savage's theory is not a descriptive or predictive theory of behavior. It is a theory which purports to advise any one of its believers how he *should* behave in complicated situations, *provided* he can make choices in a coherent manner in relatively simple, uncomplicated situations.

The fact that most people can be shown to be inconsistent in their manifest choice behavior cuts two ways: First, it emphasizes the difficulties encountered in putting into practice a model which demands in each application that the decision-maker assign a set of preferences to a host of simple problems which are internally consistent. Second, it clearly demonstrates how important it is to have a theory which can be used to aid in the making of decisions under uncertainty. If most people behaved in a manner roughly consistent with Savage's theory then the theory would gain stature as a descriptive theory but would lose a good deal of its normative importance. We do not have to teach people what comes naturally. But as it is, we need to do a lot of teaching; and to help point out this need, I particularly like the examples cited in Ellsberg's and Fellner's papers.

1. Op. cit., p. 646.

Several years ago, Ellsberg tested some of his observations on me. As an experimental subject I was hopelessly contaminated because I had already thought hard about the discussion in Jimmy Savage's book on the Allais Paradox. My immediate impulse was to break apart some of the options he asked me to consider into simpler components and then to force internal consistency on myself by means of the Savage model. But as Ellsberg pointed out, this was not playing *his* game and he asked me for a reply to his questions without any pencil pushing. I complied and I was found wanting. I was inconsistent. But I was uncomfortably inconsistent, because some of the options he posed seemed too complicated to me and I wanted to reduce them by means of the Sure-thing Principle. Of course, if I had done this, I would have forced consistency on myself. Naturally, when he pointed out that I was inconsistent I insisted that if I were playing the game for real I would pay a premium to be allowed to change my choices so that I would not violate the Sure-thing Principle. Each of us went away from that meeting with different messages: he with the notion of the limitations of the Savage theory; I with renewed respect for the importance of the Savage theory.

I started to experiment on my own with adaptations of Ellsberg's counter-intuitive examples. My subjects were students at the Harvard Business School and a few seasoned business executives — "men of experience." Immediately I observed what I shall call the "two-shift-effect." I found that when relative frequencies or so-called objective probabilities were given in numerical form as data of a decision problem, then these were often used in computing various indices (e.g., expected or actuarial values) which served as a guide to action. But if certain uncertainties in the problem were in cloudy or fuzzy form, then very often there was a shifting of gears and no effort at all was made to think deliberately and reflectively about the problem. Systematic decomposition of the problem was shunned and an over-all "seat of the pants" judgment was made which usually graphically reflected the temperament of the decision-maker. (I refer here to the pessimism-optimism polarity.) In reporting this, I am just confirming the experience of Ellsberg and Fellner (and also

of Shackle, Knight, etc.). However, I draw a different message from this experience. There is a need to teach people how to cope with uncertainty in a purposive and reflective manner, and to break down the taboo that probabilities should only be assigned if one has clear-cut relative frequency data at hand. As *one* step toward making this view more palatable I have regularly used in my classes the following adaptation of one of Ellsberg's thought-provoking problems.

I believe the following questionnaire (two questions plus the brief preamble) should be clear enough. You might enjoy trying this yourself now. In the classroom I usually emphasize the fact that "You," the subject, *have complete freedom* of calling "red" or "black" in both of the questions below.

Exhibit

An Experiment Involving Objective and Subjective Probabilities

Please answer the following questions as fairly as you can, indicating how *you personally* would act if faced with the choices described. *This is not a quiz;* there is no single "right" answer to either question.

Question 1

Suppose you have been offered the following proposition: You will be given an urn containing exactly 50 red balls and 50 black balls; there are no balls of other colors in the urn. You will name a color (red or black), and then without looking, draw a single ball from the urn. Suppose, further, that the pay-offs are as follows:

(a) If the color *you* name differs from *your* drawing you get nothing.

(b) If the color *you* name agrees with *your* drawing your gain is \$100.

Keeping in mind your financial position as of today, and remembering that the game is to be played just once, up to how much would you be willing to pay in order to participate in this game? \$——

Question 2

Suppose now, that you have been offered the same proposition as above, except that in this case the urn contains an *unknown* number of red balls and an unknown number of black balls; that is, the urn may contain all black balls, all red balls, or a mixture of the two, but there are no balls of other colors in the urn. The procedure and the pay-offs are exactly the same as in Question 1.

Keeping in mind your financial position as of today, and remembering that the game is to be played just once. up to how much would you be willing to pay in order to participate in this game? \$——

The answers for the first question clustered somewhere around \$30. Several subjects go down as low as \$10 and a few go as high as \$45. Of course, although the actuarial or expected value of the game is \$50 there is nothing "incorrect" about any of these answers. A *majority* of the subjects offered considerably more for question 1 than for question 2. For example, an answer of \$35 for question 1 and \$5 for question 2 would be a typical pair. On the other hand, a minority gave like amounts: \$40 and \$40 or \$10 and \$10, etc. According to the Savage Axioms the second option should be worth at least as much as the first. It's fun to listen to a group of subjects argue in class whether or not the second option is worse than the first. There is not only division of opinion but opinions do not change easily. But then someone — all too often that someone is I — comes up with the following argument: Suppose you withdraw a ball from the urn with unknown composition but do not look at its color. Now toss a fair (unbiased) coin and call "red" if heads, "black" if tails. The "objective" probability of getting a match is now .5 and therefore it is just as desirable to participate in the second game as in the first. I have found out that after the student convinces himself it does not matter whether the ball is drawn first or whether the coin is tossed first, that he is most willing to increase his price for the second game up to the price he was willing to pay in the first game. Incidentally, I tried this same experiment on a graduate class in statistics in which all the students had prior courses in mathematical statistics and the pattern of answers, as well as the ensuing free-for-all discussion, was very much like the experiences I had with business school students who had had no previous courses in statistics.

As a second example let us use the very ingenious example given by Ellsberg, where a subject chooses a ball from an urn containing 30 red balls and 60 (black and yellow) balls, the latter in unknown proportion. Recall that the four acts he considers lead to the array:

		State		
		Red	Black	Yellow
Act	I	100	0	0
	II	0	100	0
	III	100	0	100
	IV	0	100	100

I concur with Ellsberg that most subjects choose act I over II and IV over III, thus violating the Savage Axioms. Also I admit that many of these subjects are reluctant to change their choices when it is pointed out to them that their behavior is inconsistent with the Sure-thing Principle. Many are also not very impressed with the argument that *no* partition of the sixty (black, yellow) balls would lead to the pattern I over II and IV over III. Well, I would like to undermine further their confidence in their initial choices! I find the following argument is quite persuasive. Suppose you register I > II and IV > III. I now offer you the paired comparison between the following two options: In option A a fair (unbiased) coin is tossed and act I is taken if heads appears, whereas act IV is taken if tails appears; with option B, heads leads to act II and tails to act III. In table form we get:

	Heads	Tails
Option A:	Act I	Act IV
Option B:	Act II	Act III

Now by strict dominance option A is better than B *for you* since you prefer I to II and IV to III. So far there is no trouble but let's take a closer look at options A and B. The final outcomes of either option depend on the toss of the coin and the selection of a ball. Now let's do the accounting by analyzing the implications of the options conditional on the color of the withdrawn ball. Our analysis takes the form:

	Red	Black	Yellow
Option A	An "objective" 50-50 chance of \$100 and \$0	→ same	→ same
Option B	↓ same	→ same	→ same

But this reasoning should lead everyone to assert that options A and B are *objectively identical!* Something must give! I cannot see how anyone could refute the logic leading to the conclusion that given your initial choices, you should prefer option A to option B. This bit of logic is certainly not the weak link in the argument. But then again these options look awfully alike to me! Therefore, on thinking it over, wouldn't you like to change your mind about your initial preferences?

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